applied optics

Icebows

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Theoretical and experimental studies show that water ice spheres can produce a rainbow in which the primary and secondary bows overlap. To our knowledge, no such natural "icebow" has ever been reported.

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1. INTRODUCTION

The geometrical optics of rainbows are well understood, and have been since Descartes first ray traced them [1]. The wave theory of rainbows has also been solved [2,3]. Yet much less attention has been paid to rainbows produced in substances other than water. Seawater is known to produce rainbows at scattering angles that are slightly different than that of pure water [4,5]. Some work has also gone into possible rainbows on other planets, for example those produced by H_2O, CH_4 and H_2SO_4, each of which is known to exist in liquid form in the solar system [6,7].

While investigating rainbows from spheres with different indices of refraction, we noticed (as had others [8,9]) that when the refractive index n is near 1.31, the primary and secondary rainbows nearly overlap (Fig. 1). Since water ice's index is about 1.31, it seemed that transparent ice spheres or cylinders could produce an unusual rainbow, an *icebow*.

Glare points are minimum deviation caustics in transparent cylinders that are analogous to rainbow caustics. Marston [9] remarks in his study of glare points in dielectric cylinders (e.g., icicles), "For a sufficiently small value of [the inclination angle] corresponding to nearly horizontal illumination, the model suggests that a sufficiently distant observer could lie within the two-ray region of both the primary and the secondary caustics." There is also tantalizing evidence—both photographic [8] and lidar remote sensing [10,11]—that suggests the possibility of icebows from frozen raindrops and "quasi-spherical" ice particles.

To our knowledge, no naturally occurring icebow has ever been observed and verified to originate from ice spheres. We caution the reader that in popular scientific literature, the term "icebow" has been frequently used to describe ice crystal halos in the shape of a "bow" and not to rainbows formed in ice spheres. Strictly speaking, only a cloud of ice spheres can produce an icebow.

In this paper, we present a classical ray optics analysis of ice spheres and show laboratory photographs that demonstrate primary and secondary rainbow rays emerging parallel to one another in ice spheres and cylinders. If a cloud of either shape (randomly oriented for cylinders) were viewed from a great distance, an icebow would be seen.

2. REQUIREMENT FOR PRECISE OVERLAP

As is well known [3,12], the angle of incidence i of light striking a sphere that leads to minimum deviation with k internal reflections is

$$\cos i = [(n^2 - 1)/k(k+2)]^{1/2}. \quad (1)$$

The angle of minimum deviation D in degrees is then found from

$$D = 180°k + 2i - 2(1+k)r, \quad (2)$$

where r is the angle of refraction corresponding to i from Snell's law. Numerically, both i and r are measured in degrees. Defining D_1 and D_2 to be the minimum deviation angles for the primary and secondary bows (Fig. 2), the scattering rays from the two bows will be parallel providing that the following conditions are true:

$$D_1 + f = 180°, \quad (3)$$

$$D_2 - f = 180°, \quad (4)$$

where f is an angle that must be the same for the two bows to overlap. By adding Eqs. (3) and (4), we eliminate f and find the requirement for perfect coincidence, namely,

$$D_1 + D_2 = 360°. \quad (5)$$

Dispersion aside, this condition assures that the primary and secondary bows will leave the drop traveling in the same direction, i.e., parallel to each other, though emerging from opposite

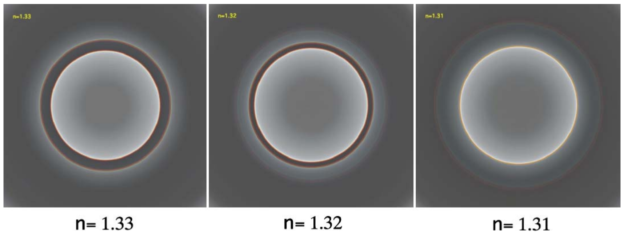

Fig. 1. Rainbow simulations centered on the antisolar point. For water ($n = 1.33$), the familiar primary and secondary bows are evident with Alexander's dark band between them. As the index is reduced, the two bows come closer together until when $n = 1.31$, they overlap. When $n < 1.31$, the secondary bow is inside the primary bow.

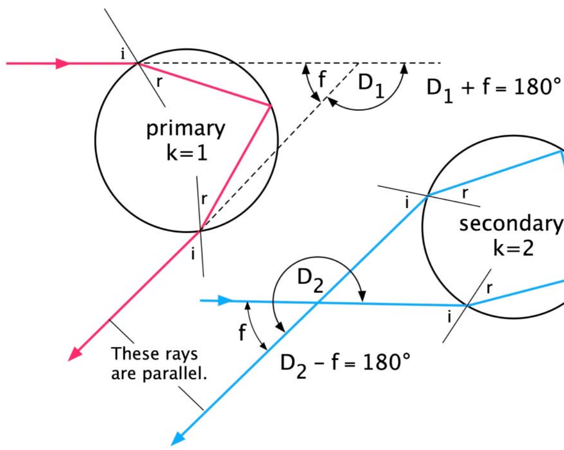

Fig. 2. Condition for perfect overlap is that $D_1 + D_2 = 360°$, where D_1 and D_2 are the minimum deviation angles of the primary and secondary bows, respectively.

sides of the drop. From the observer's view point, the scattering angle would be 134.8°, essentially 135°, and seen at 45° from the antisolar point.

By evaluating D_1 and D_2 numerically, we found that $n = 1.3120125\ldots$ satisfies Eq. (5) to seven significant figures. For water ice, this index corresponds to a wavelength ~530 nm, squarely in the green part of the spectrum [13], and therefore is easily accessible to human vision.

To a distant observer, both bows would appear as a single bow because the light from them would be coming from the same direction. The colors would be dominated by the much brighter primary bow. With the primary and secondary's dispersions being oppositely directed, the icebow's colors would be further muted.

3. LABORATORY EXPERIMENTS

Ice spheres and cylinders with a diameter of about 4.3 cm were made in a freezer and illuminated with collimated white light from an LED flashlight in a goniometer system on an optical bench. The distances between the observer and the ice ball, and the light source and the ice ball were both approximately 6 m. The observer, sphere, and illumination source were all in a horizontal plane. The ice ball subtended an angle of 0.4°, an amount smaller than the width of either bow. Being close enough to the sphere to resolve it allowed us to move around the sphere and watch each rainbow ray come and go. Observing from a large but not infinite distance meant that the bow rays coming from the ball were not precisely parallel, but dispersion allowed us to identify each bow unambiguously.

Both rainbow rays were found at the expected scattering angle, 135° (Fig. 3). Because of the additional internal reflection experienced by secondary rays, the two rays emerged from opposite sides of the sphere, and therefore could be seen as separate bright spots on the sphere. Surface irregularities caused by the melting ice distorted the rainbows but the bows were nonetheless unquestionably present. This was verified as the bows appeared and disappeared when the observer's line of sight (i.e., scattering angle) changed.

As expected, the primary bow in Fig. 3 is much brighter than the secondary bow. The reasons that the primary appears red and the secondary is blue are both experimental in nature and intrinsic to the bows. The observer was at a large distance (compared to the diameter of the sphere) but not an infinite distance from the sphere, so the precise conditions that would prevail for a natural icebow were not strictly met. The photograph was selected from many to best show the two bows. A slight shift in observer position would cause the bows to change color and brightness as a result of the reversed dispersion, so we selected a photo which seemed to best show the phenomenology.

When the sphere is resolved, the two bows are spatially separated from one another and do not "overlap" in the sense that they come from two different locations on the sphere. But for a distant observer where the sphere is not resolved, the direction from which the two bows comes would be the same and thus they would overlap.

To check if both spots on the ice ball were the two sought after rainbow rays, other spheres were examined under

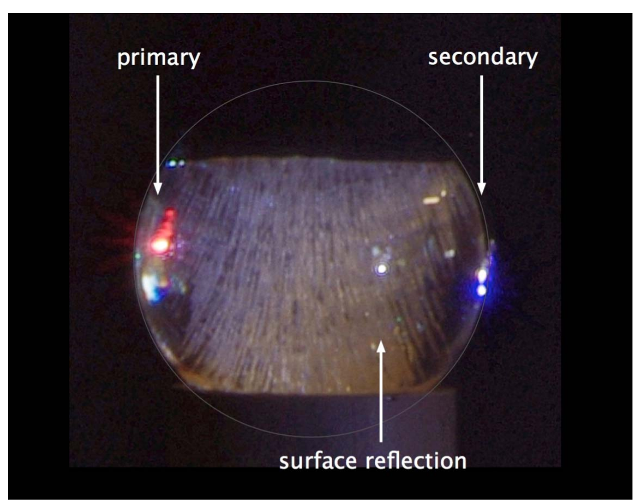

Fig. 3. Ice sphere observed at a scattering angle of 135°. The primary and secondary bows are evident on opposite sides of the sphere. Irregularities in the surface distort the bows. Ice sphere was 4.3 cm in diameter and viewed from a distance of about 6 m. Illumination was from a collimated white-light LED flashlight.

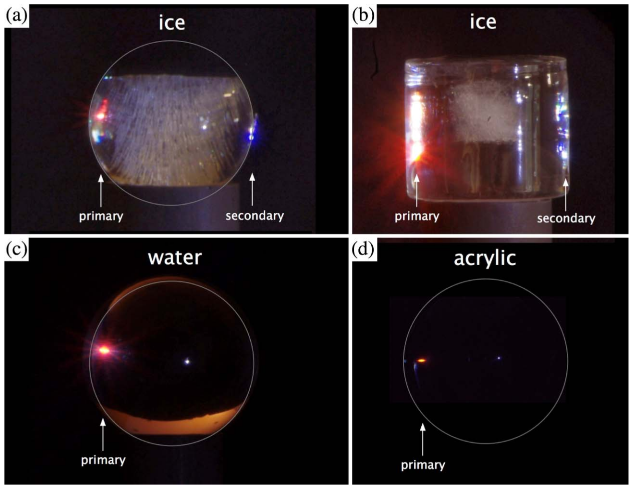

Fig. 4. (a) Ice sphere viewed at a scattering angle of 135°. The primary and secondary bows are evident, as they are in an ice cylinder (b). (c), (d) Water and acrylic, respectively, viewed from the angle necessary to see the primary bow. As expected, the secondary bows are absent because they emerge from the spheres at angles that are significantly different than the scattering angles of their primary bows. The water sphere (c) was a thin-walled hollow glass sphere filled with water, and the orange glows are due to ambient room lights. In (a), (c), and (d), the outlines of the spheres have been drawn in.

the same circumstances (water, acrylic, and glass, $n = 1.33$, 1.48, and 1.52, respectively). Only in the case of ice were both bow rays visible from a single location. Specifically, when viewed from a direction showing the primary bow, the secondary bows were not seen [Figs. 4(c) and 4(d)]. This is to be expected because the spheres had refractive indices that were very different than 1.31, and thus the primary and secondary rays would not be parallel.

4. DISCUSSION

It is not surprising that ice spheres can produce rainbows. But it is interesting that ice has exactly the right refractive index to cause the primary and secondary bows to overlap. This happens because both bows emerge from the drop at the same scattering angle.

To form natural icebows, one needs a cloud of spherical or cylindrical ice particles larger than a few tens of micrometers in size. Such particles would need to be well separated from one another and illuminated by the sun. Possible places to look are in mixed phase clouds, sleet, hail, old snow, and icicles. Even ice cylinders that are tapered or did not have a circular cross section would produce glare points but they would rarely if ever produce both icebow glare points as seen from the same observer location.

There is abundant evidence in the cloud community that "quasi-spherical" or spherical ice particles exist in some cold clouds [14–17]. The claim that the particles are made of ice is based on *in situ* collection methods and lidar measurements showing no polarization in backscatter. Visual observations of clouds that "look like cirrus" have shown cloud coronae, though coronae can also be formed by small nonspherical particles. No rainbows have been reported in these clouds and it is not known if the particles were transparent.

Icicles offer a more accessible chance to see icebow glare points. They are numerous and transparent. Observing the bow is simply a matter of finding icicles with circular cross sections that are illuminated by the sun in such a way that the observer can view them 45° from the antisolar point. To our knowledge, no such natural icebow glare points have ever been reported. It seems likely, however, that casual observers have seen icebow glints in icicles but have not recognized them as being significant or interesting.

As a final note, observers should take care to measure the image scale on their photographs, and then measure the radii or other defining feature of rainbows and ice crystal halos. Odd radius halos are often only a degree or two different than the common halos, and such small angular distance could pass unnoticed by eye. This is especially true for an icebow, which is only 3 deg larger than the primary rainbow.

5. SUMMARY AND CONCLUSIONS

Theory and experiments reported here show that overlapping primary and secondary rainbows occur in ice spheres and ice cylinders. When viewed from great distances, the two bows would overlap, creating an icebow.

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