Optics of sunbeams

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Crepuscular and solar rays are visible by means of the contrast between sunlit and (usually) cloud-shaded portions of the atmosphere. Their visibility depends on (1) the volume angular-scattering coefficient $\beta(\phi)$, (2) the brightness of the sunlit sky, and (3) the integrated optical path through the shadowed regions of the atmosphere. We show that the geometry of path length through the umbra (3) is sufficiently important that it can account for most of the observed properties of rays, except when the volume angular-scattering coefficient $\beta(\phi)$ is sharply peaked in the forward direction.

1. INTRODUCTION

"Sunbeams" is the colloquial name for rays formed by the shadows of scattered or broken clouds seen in the Earth's atmosphere. Other common names are "crepuscular rays," "Buddha's fingers," "ropes of Maui," "sun drawing water," and "backstays of the Sun." If the ray is formed when the Sun shines through a gap in the clouds it is bright. When a relatively isolated cloud casts its shadow on the sky the ray appears dark. In both instances, scattering particles are necessary to render the ray visible. The scatterers may be dust, snow, rain, or simply air molecules acting as Rayleigh scatterers. They are visible by means of the contrast between sunlight scattered by the atmosphere and skylight scattered in the umbra of a cloud. The usual appearance of rays is of one or more straight shafts of light that appear to diverge from the Sun and that are sometimes seen opposite the Sun as rays converging to the antisolar point. All the shadows are parallel, but, owing to the small distance of the observer to the rays (compared with his distance from the Sun), perspective makes the rays appear to diverge.

There is surprisingly little quantitative information about rays. Indeed, the only scientific literature known to this author is among popular scientific treatises (e.g., papers by Minnaert,[1] Meinel and Meinel,[2] Greenler,[3] and Montheith,[4] although some of their geometrical properties are similar to those of mountain shadows.[5,6] In this paper we present a first-order analysis of the contrast of rays and show that many aspects of their visibility are independent of the scattering mechanism and can be explained purely by the geometry of shadows.

2. OBSERVATIONS

Rays are neither uniformly visible over the entire sky nor as likely to be seen at any time throughout the day. Careful study of rays reveals the following properties:

(1) Rays are most common when the Sun is low.
(2) Rays are most common when the air is clear.
(3) The rays' greatest visibility (contrast) occurs within 15° of the Sun and the antisolar point.
(4) Rays near the antisolar point reach their maximum visibility within a few degrees of the antisolar point.
(5) Antisolar rays are usually associated with large clouds overhead or near the antisolar point and are seldom observed from small clouds, unless the cloud is near the antisolar point.
(6) Maximum intensity contrast occurs near the Sun, and maximum color contrast occurs near the antisolar point.

In Section 3 we shall attempt to explain these observed characteristics of rays in terms of the factors that contribute to the ray's contrast.

3. THEORY

We shall assume throughout the following discussion that the line of sight is optically thin, i.e., that the brightness is directly proportional to the number of scatterers. Referring to Fig. 1, consider a unit volume, whose volume angular-scattering coefficient is $\beta(\phi)$, located a distance r from observers positioned at points A through G. The Sun is on the horizon or slightly above it (elevation, ~0.0). The observer looking through unshadowed air receives an amount of light E_b:

$$E_b = \int_0^\infty E_\odot \beta(\phi) \mathrm{d}r = \int_0^{r_1} E_\odot \beta(\phi) \mathrm{d}r + \int_{r_1}^{r_2} E_\odot \beta(\phi) \mathrm{d}r + \int_{r_2}^{\infty} E_\odot \beta(\phi) \mathrm{d}r, \quad (1)$$

where $E_\odot$ is the solar illuminance and ϕ is the scattering angle. In this case ϕ is also the elevation angle of the observer's line of sight. The integral is calculated over the entire path through the atmosphere. E_b is a strong function of solar elevation ϕ, especially near the horizon where the air mass increases rapidly.[7]

Some first-order insights into ray contrast can be obtained by analyzing a ray formed by a cloud whose vertical thickness is T_c and whose cloud base is y_c. Let the Sun illuminate the cloud located at a horizontal distance x_c from the observer (at points A through G). We shall chose a cloud whose base height is 2 km and whose vertical thickness is 1 km, both of which are sufficiently small compared with the density scale height of the atmosphere (~8.5 km) that vertical variations in the scattering parameters can be safely ignored.

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The locations of the observers at points A through G are x_c = $1.25U$, $1.0U$, $0.75U$, $0.50U$, $0.25U$, $0.0U$, and $-0.25U$, respectively. The length of the umbra U is

$$U = T_c \times 57.3/0.5,$$

where 0.5 in the above expression represents the half-degree diameter of the Sun. In this example U = 115 km. The brightness of the sky along a line of sight through the umbra is the ray brightness E_r:

$$E_r = \int_0^{r_1} E_\odot\beta(\phi)\mathrm{d}r + \int_{r_1}^{r_2} S\,\mathrm{d}r + \int_{r_2}^{\infty} E_\odot\beta(\phi)\mathrm{d}r. \qquad (2)$$

The second term represents skylight scattered in the umbra to the observer. Obviously, S is itself the integral

$$S = \int_0^{2\pi}\int_0^{\pi/2} E(\theta,\varphi)\beta[\phi'(\theta,\varphi)]\sin\theta\,\mathrm{d}\theta\mathrm{d}\varphi,$$

where $E(\theta, \varphi)$ is the brightness of the sky illuminating the shadow, as a function of the polar angles θ and φ, and $\beta(\phi')$ is the same function as $\beta(\phi)$, the primes indicating that the scattering angle is not the elevation of the observer's line of sight. The contrast C of the ray is defined as

$$C = (E_r - E_b)/E_b.$$

The contrast of the ray is determined by the brightness of the sky viewed through the shadow as compared with the brightness of the fully illuminated sky. To first order, let us consider the contribution to the contrast of the ray as being due only to those regions of space occupied by the umbra of the shadow and compare it with the brightness of the sky in a completely unshadowed region. The expression for the contrast can be simplified by noting that the first and third terms in Eqs. (1) and (2) are identical and thus

$$C = \frac{\int_{r_1}^{r_2}(S - E_\odot\beta)\mathrm{d}r}{\int_0^{\infty} E_\odot\beta\,\mathrm{d}r} \simeq \frac{(S - E_\odot\beta)(r_2 - r_1)}{\int_0^{\infty} E_\odot\beta\,\mathrm{d}r}.$$

For an optically thin atmosphere $r_2 - r_1 = D$. Clearly, C is proportional to $(S - \beta E_\odot)$ and D, the path through the umbra. In general, all three quantities will depend on ϕ. A simple analysis reveals that, for positive (i.e., realistic) values of D,

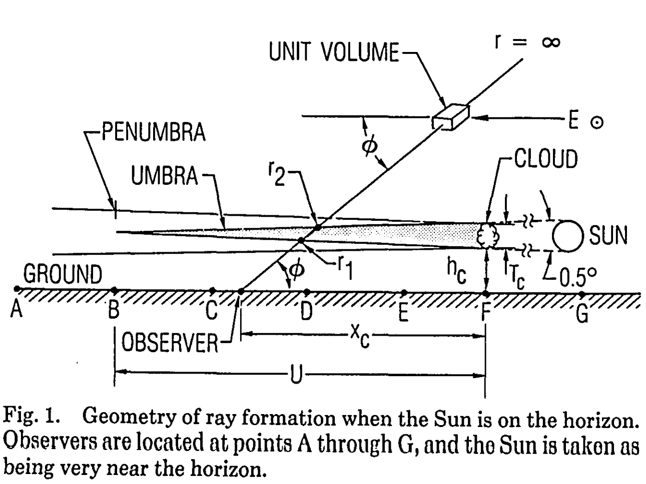

Fig. 1. Geometry of ray formation when the Sun is on the horizon. Observers are located at points A through G, and the Sun is taken as being very near the horizon.

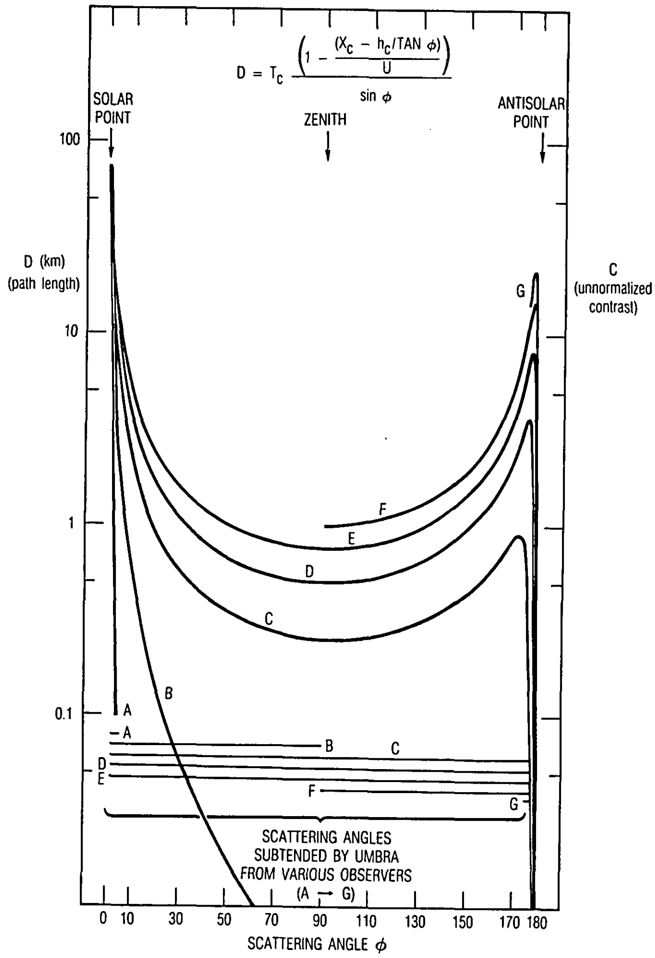

Fig. 2. Path length D (in kilometers) through the umbra (stippled region in Fig. 1) as a function of scattering angle ϕ (in degrees) for various observers located at positions A through G. Parameters T_c = 1 km, h_c = 2 km, Sun elevation = 0°, horizontal cloud distance x_c = $1.25U$, $1.00U$, $0.75U$, $0.50U$, $0.25U$, $0.0U$, and $-0.25U$, where U is the length of the umbra, in this case 115 km.

$$D(\phi) = T_c\left[- \frac{(x_c - b_c/\tan\phi)}{u}\right]\csc\phi, \qquad (3)$$

where the first term is the vertical thickness of the umbra and the second term $[\csc(\phi)]$ scales the vertical thickness by the slant path through the umbra at a line-of-sight elevation of ϕ.

If β is not a strong function of ϕ, which it is not for Rayleigh scattering, then C is proportional to D, the geometrical line-of-sight distance through the umbra. When $\beta(\phi)$ displays strong forward and backward scattering, as in the case of dust particles, C is further enhanced in these directions. $D(\phi)$ is shown in Fig. 2 for each of the observers located at points A through G in Fig. 1. $D(\phi)$ represents the unnormalized contrast of the ray.

4. DISCUSSION

A number of features are quickly deduced from Fig. 2. Ray contrast is higher near the solar and antisolar points because the line of sight through the umbra, which scales roughly as csc(ϕ), is greatest near 0° and 180°. As defined in Eq. (3), D would go to infinity at the extreme scattering angles, but in practice it does not; $\phi = 0°$ is never reached because the cloud is not infinitely far away, and $\phi = 180°$ occurs beyond the location where the umbra ceases to exist because U is finite. The sharp drop in contrast that reaches a broad minimum near the zenith ($\phi = 90°$) is also due primarily to the csc ϕ term in Eq. (3).

With the calculated properties of $D(\phi)$ in mind, let us recall the observed characteristics of rays listed in Section 2 and examine them with respect to $D(\phi)$.

1. *Rays Are Most Common When the Sun Is Low.* If the Sun is high, the umbra (length U) intersects the ground and effectively shortens it to the point that D will also be correspondingly shortened.

2. *Rays Are Most Common When the Air Is Clear.* Clearly, if the sky is hazy, $E_{\odot}$ is low, and absorption may diminish the ray's contrast.

3. *The Rays' Greatest Visibility (Contrast) Occurs Within 15° of the Sun and the Antisolar Point.* This point has been explained in the first paragraph of this section.

4. *Rays Near the Antisolar Point Reach Their Maximum Visibility Within a Few Degrees of the Antisolar Point.* This point has also been explained in the first paragraph of this section.

5. *Antisolar Rays Are Usually Associated with Large Clouds Overhead or Near the Antisolar Point and Are Seldom Seen from Small Clouds, Unless the Clouds Are Near the Antisolar Point.* Figure 2 shows that, if the umbra of the cloud does not reach very near the antisolar point, as with a small cloud, there will be no ray seen here. A large cloud may have a long umbra that may reach to the antisolar point. Owing to the length of the umbra, an intermediate cloud may have an antisolar ray if it is overhead. Note that maximum contrast for a ray overhead occurs when the cloud is nearly overhead (observers located at points E and F in Fig. 2); this is because the umbra is thickest here. Owing to the high contrast of rays near the antisolar point (observer at point G in Fig. 1), even a tiny cloud can cause a short, high-contrast ray.

6. *Maximum Intensity Contrast (C) Occurs Near the Sun, and Maximum Color Contrast Occurs Near the Antisolar Point.* The color contrast can not be explained solely in terms of $D(\phi)$; it is caused by the scattering mechanism and the illumination of the umbra. Near the antisolar point the ray may be seen against the pinkish Belts of Venus. Since the umbra is illuminated by the blue sky and therefore appears bluish, the umbra and background sky are near opposite ends of the spectrum and thus display moderately strong color contrast. In the direction of the Sun, the sky tends to be whitish or whatever color that the Sun appears to be, owing to scattered light from the aureole. The rays, especially when they are due to dust particles, strongly forward scatter light from the whitish skylight; thus the color contrast is low.

5. CONCLUSION

We have compared the observed properties of rays with their theoretical contrast based on a line-of-sight calculation. To first order, the contrast of rays is dominated by the line-of-sight path length through the umbra. Furthermore, their contrast is often independent of the scattering mechanism producing them.

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